

Ch 2.4 P1

MA 121

May 23 18

$$(5) \lim_{x \rightarrow \infty} \frac{2+x^5}{5x^6-2x}$$

$$= \lim_{x \rightarrow \infty} \frac{x^5 \left(\frac{2}{x^5} + 1 \right)}{x^6 \left(5 - \frac{2}{x^5} \right)}$$

← factor out highest power in numerator (x^5)

$$= \lim_{x \rightarrow \infty} \left(\frac{1}{x} \left(\frac{\frac{2}{x^5} + 1}{5 - \frac{2}{x^5}} \right) \right)$$

$$= \left(0 \left(\frac{0+1}{5-0} \right) \right)$$

$$= (0) \left(\frac{1}{5} \right) = 0$$

$$\therefore \lim_{x \rightarrow \infty} \frac{2+x^5}{5x^6-2x} = 0$$

$$(10) \lim_{x \rightarrow \infty} \frac{x+4}{\sqrt{25x^2+1}}$$

$$= \lim_{x \rightarrow \infty} \frac{x \left(1 + \frac{4}{x} \right)}{\sqrt{x^2 \left(25 + \frac{1}{x^2} \right)}}$$

$$= \lim_{x \rightarrow \infty} \frac{x \left(1 + \frac{4}{x} \right)}{\sqrt{x^2} \sqrt{25 + \frac{1}{x^2}}}$$

$$= \lim_{x \rightarrow \infty} \frac{x \left(1 + \frac{4}{x} \right)}{x \sqrt{25 + \frac{1}{x^2}}}$$

$$= \lim_{x \rightarrow \infty} \frac{1 + \frac{4}{x}}{\sqrt{25 + \frac{1}{x^2}}}$$

$$= \frac{1+0}{\sqrt{25+0}}$$

$$= \frac{1}{\sqrt{25}}$$

$$= 1/5$$

← $\sqrt{x^2} = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$
 when $x \rightarrow \infty$, we have $x \geq 0$
 therefore $\sqrt{x^2} = x$

Hibroy

$$(ii) \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 3}}{10x}$$

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 \left(1 + \frac{3}{x^2}\right)}}{10x}$$

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2} \sqrt{1 + 3/x^2}}{10x}$$

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2} \sqrt{1 + 3/x^2}}{10x}$$

$$= \lim_{x \rightarrow -\infty} \frac{-x \sqrt{1 + 3/x^2}}{10x}$$

$$= \lim_{x \rightarrow -\infty} \frac{-\sqrt{1 + 3/x^2}}{10}$$

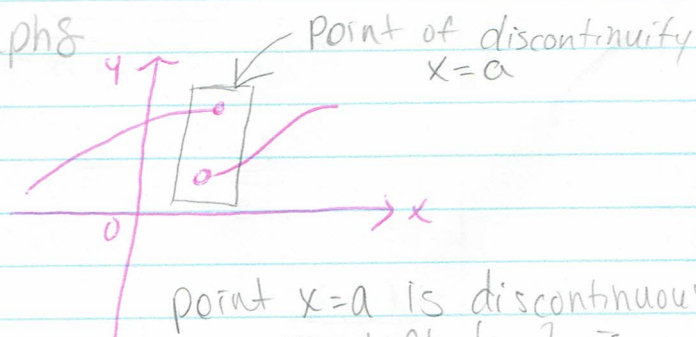
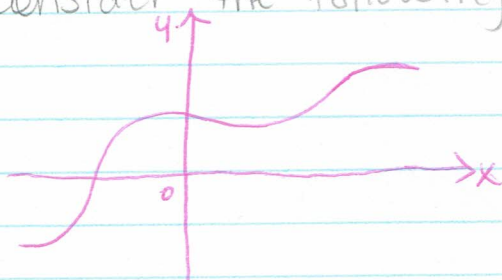
$$= \frac{-\sqrt{1 + 0}}{10}$$

$$= -\frac{1}{10}$$

When $x \rightarrow -\infty$, we have $x < 0$
therefore $\sqrt{x^2} = -x$

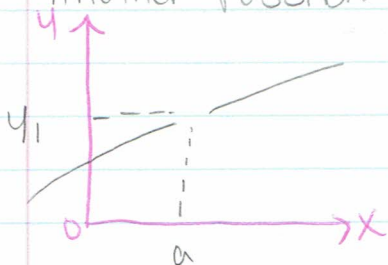
2.5 continuity

Consider the following graphs



point $x=a$ is discontinuous because the left limit is not the same as the right limit. In other words, the limit is undefined.

Another Possibility



Even though $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$

or $\lim_{x \rightarrow a} f(x)$ is defined, $\lim_{x \rightarrow a} f(x) \neq f(a)$

Since $f(a)$ may be undefined or elsewhere.

In summary, a function f is said to be continuous at $x=a$ if $(y=f(x))$

↳ $f(a)$ is defined

↳ $\lim_{x \rightarrow a} f(x)$ is defined. (ie $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$)

↳ if both of the above are satisfied and if the limit $\lim_{x \rightarrow a} f(x) = f(a)$

(1) p62

$f(x) = 5 - 8x$ at $x = -10$ ← check 3 conditions.

is it defined at $x = -10$?

$f(-10) = 5 - 8(-10) = 85$. it is defined.

(2) $\lim_{x \rightarrow -10} (5 - 8x) = 5 - 8(-10) = 85 \therefore \lim_{x \rightarrow -10}$ is defined.

(3) We see that

$\lim_{x \rightarrow -10} f(x) = f(-10) \therefore f(x) = 5 - 8x$ is continuous at $x = -10$
when we draw the graph of $y = 5 - 8x$
pen is not lifted at $x = -10$

(2p62) $f(x) = \frac{1}{x}$ at $x = 0$

i) $f(0) = \frac{1}{0}$ which does not exist.

$\therefore y = \frac{1}{x}$ is discontinuous at $x = 0$

(5PQ2) $f(x) = \begin{cases} x+1 & \text{if } x \leq 3 \\ x^2-5 & \text{if } x > 3 \end{cases}$
at $x=3$

i) $f(3) = 3+1=4$ which is defined.

ii) $\lim_{x \rightarrow 3^-} f(x) =$

$= \lim_{x \rightarrow 3^-} (x+1)$

$= 3+1$

$= 4$

and $\lim_{x \rightarrow 3^+} f(x) =$

$= \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (x^2-5)$

$= 3^2 - 5 = 9-5$

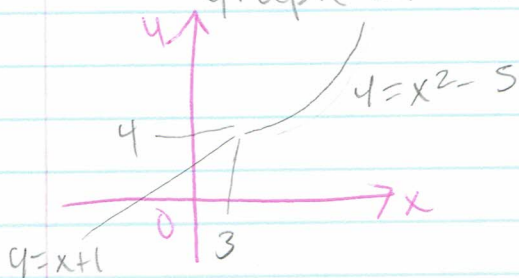
$= 4$

$\therefore \lim_{x \rightarrow 3} f(x) = 4$

We see that

$\lim_{x \rightarrow 3} f(x) = f(3) = 4$ $\therefore f(x)$ is continuous at $x=3$

graph of $f(x)$:



(6P52)

$f(x) = \begin{cases} x^3+1, & \text{if } x < -2 \\ -x^2, & \text{if } x \geq -2 \end{cases}$
at $x=-2$

i) $f(-2) = -(-2)^2 = -4$

ii) $\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^-} (x^3+1) = (-2)^3 + 1 = -8+1 = -7$

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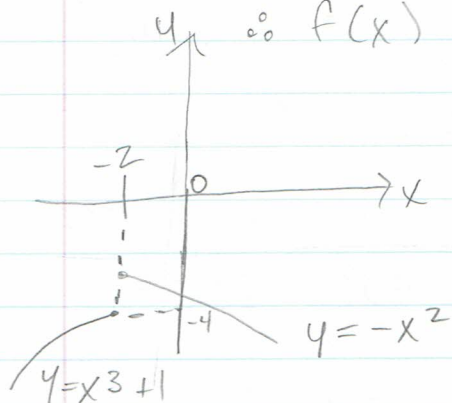
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and $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (-x^2) = -(-2)^2 = \boxed{-4}$

Since $\lim_{x \rightarrow -2^-} f(x) \neq \lim_{x \rightarrow -2^+} f(x)$,

$\lim_{x \rightarrow 2} f(x)$ is undefined.

$\therefore f(x)$ is discontinuous at $x = -2$



(18 p62) Locate all discontinuities.

$$f(x) = \begin{cases} \frac{15}{x+2}, & \text{if } x < 1 \\ 3, & \text{if } x \geq 1 \end{cases}$$

at $x=1$,

i) $f(1) = 3$

ii) $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \left(\frac{15}{x+2} \right) = \frac{15}{1+2}$

$= \frac{15}{3} = 5.$

$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (3) = 3$

Since $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$

$\therefore \lim_{x \rightarrow 1} f(x)$ is undefined.

$f(x)$ is discontinuous at 1

And notice that $f(x) = \frac{15}{x+2}$ when $x < 1$

Hilroy

It is discontinuous when $x+2=0$; $x=-2$

∴ there are two points of discontinuity for $f(x)$, i.e. at $x=-2$ and $x=1$.

(21p63)

We want to find the value of c that fulfills all 3 conditions for continuity,

$$\text{i.e.) } \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

So, we have

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2) \quad f(x) = \begin{cases} cx^2, & \text{if } x < 2 \\ cx + 20, & \text{if } x \geq 2 \end{cases}$$

$$\lim_{x \rightarrow 2^-} (cx^2) = \lim_{x \rightarrow 2^+} (cx + 20) = (c)(2) + 20$$

$$= (c)(2)^2 = (c)(2) + 20 = (c)(2) + 20$$

$$= 4c = 2c + 20$$

$$= 4c - 2c = 20$$

$$= 2c = 20$$

$$c = 10$$

END NOTE. A function $y=f(x)$ is said to be continuous on an interval $x \in (a, b)$ if it is continuous on all points on $x \in (a, b)$